ADMM-Based Delay-Doppler Domain Channel Estimation for OTFS Systems

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Outline

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2. System Model
3. Proposed Method
3. Simulation Results
4. Conclusion

Introduction

- ✓ **Background**
- ✓ **Previous Works**
- ✓ **Motivations**
- ✓ **Contribution**

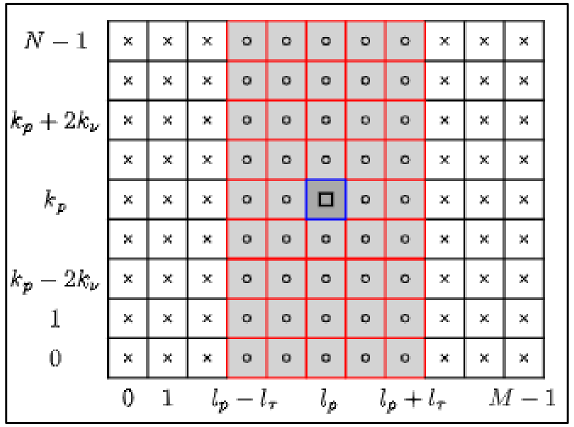
Background

- OFDM fails in high-Doppler scenarios (V2X, NTN) → ICI.
- OTFS operates in Delay-Doppler (DD) domain.
- Challenge: Accurate DD-domain channel estimation under noise and unknown sparsity.

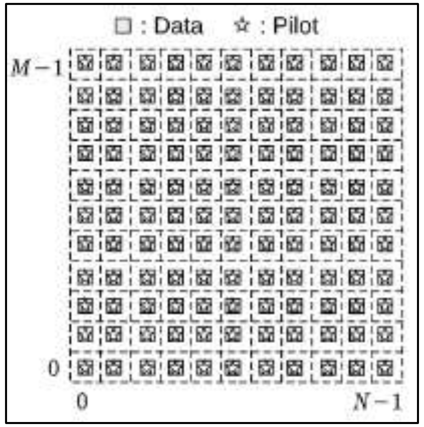
Aug, 2025: 6G meeting settles on the same old air interface.

Previous Works (1/2)

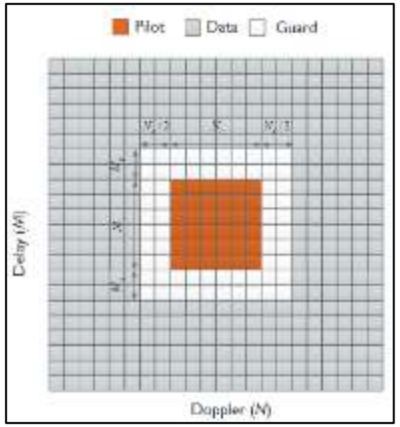
- Channel Estimation
  - Embedded pilot-aided: low spectral efficiency
  - Superimposed pilot-aided: interference-induced
  - Compressed sensing: Orthogonal matching pursuit (OMP), sparsity adaptive matching pursuit (SAMP), smoothed l_0 (SL0), …



Embedded Pilot



Superimposed Pilot



Pseudo-random Pilot

* P. Raviteja, K. T. Phan and Y. Hong, "Embedded Pilot-Aided Channel Estimation for OTFS in Delay–Doppler Channels," in IEEE Transactions on Vehicular Technology, vol. 68, no. 5, pp. 4906-4917, May 2019.
* H. B. Mishra, P. Singh, A. K. Prasad and R. Budhiraja, "OTFS Channel Estimation and Data Detection Designs With Superimposed Pilots," in IEEE Transactions on Wireless Communications, vol. 21, no. 4, pp. 2258-2274, April 2022.
* W. Shen, L. Dai, J. An, P. Fan and R. W. Heath, "Channel Estimation for Orthogonal Time Frequency Space (OTFS) Massive MIMO," in IEEE Transactions on Signal Processing, vol. 67, no. 16, pp. 4204-4217, 15 Aug.15, 2019.

Previous Works (2/2): SL0 Channel Estimation

- **Problem:** OTFS channel estimation → Sparse recovery: $\boldsymbol{y} = \boldsymbol{\Psi h} + \boldsymbol{w}$
- **SL0 idea:** approximate non-convex l_0-norm with smooth Gaussian $f_\sigma(h_i) = e^{-\frac{|h_i|^2}{2\sigma^2}}$,
  minimize $F_\sigma(\boldsymbol{h}) = \sum_i f_\sigma(h_i)$ via gradient descent
- **Advantages**
  - No prior knowledge of sparsity
  - Fast reconstruction, low complexity
- **Limitations**
  - Non-convex, may fall into local optima, no guarantee of global convergence
  - Parameter-sensitive (σ, step size μ_0)

X. Wang, C. Zheng, P. Hu, J. Yang and C. G. Kang, "A Sparsity-Agnostic SL0 Channel Estimation Approach for OTFS Systems," in *IEEE Communications Letters*, vol. 29, no. 5, pp. 1097-1101, May 2025.

Motivation and Contribution

Motivations

- DD channel exhibits sparse structure → suitable for compressed sensing.
- Existing CS methods (OMP, SL0, BP): need sparsity prior, or lack stability.

Contributions

- Keep the CS framework, but solve it convexly with Alternating Direction Method of Multipliers (ADMM).
  - ℓ_1-regularized convex optimization formulation.
  - Pilot structure satisfying incoherence.
  - ADMM-based solver: no sparsity prior, stable convergence.
  - Superior NMSE, BER, runtime tradeoff.

System Model

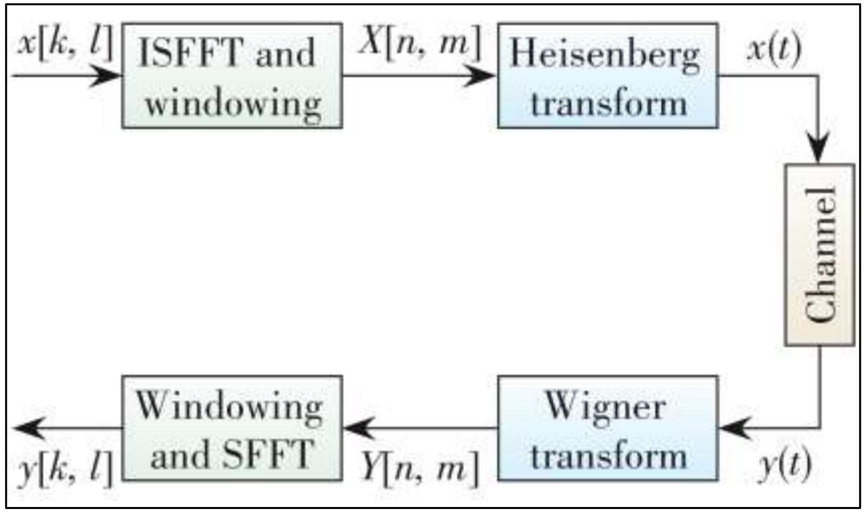
- ✓ **OTFS I/O Relationship**
- ✓ **Pilot Design**

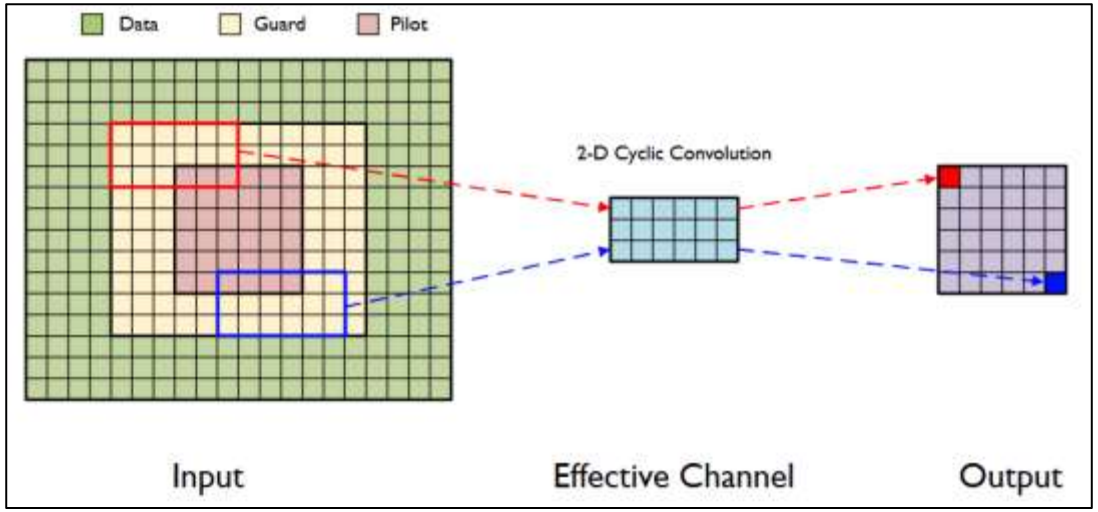
OTFS I/O and Pilot Design

- **OTFS signal chain:** DD grid → ISFFT → Heisenberg → Channel → Matched Filtering → SFFT.
- **Vectorized model:**

$$\boldsymbol{y} = \boldsymbol{\Psi h} + \boldsymbol{w}.$$

- **Pilot design:** pseudo-random pilots + guard intervals ensure incoherence.





Source: Ashwitha Naikoti, and Ananthanarayanan Chockalingam, "Signal Detection and Channel Estimation in OTFS," ZTE communications

Optimization Model and ADMM

- ℓ_1-regularized convex problem:

$$\min_{\mathrm{h}} \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{\Psi}\boldsymbol{h}\|_2^2 + \lambda\|\boldsymbol{h}\|_1.$$

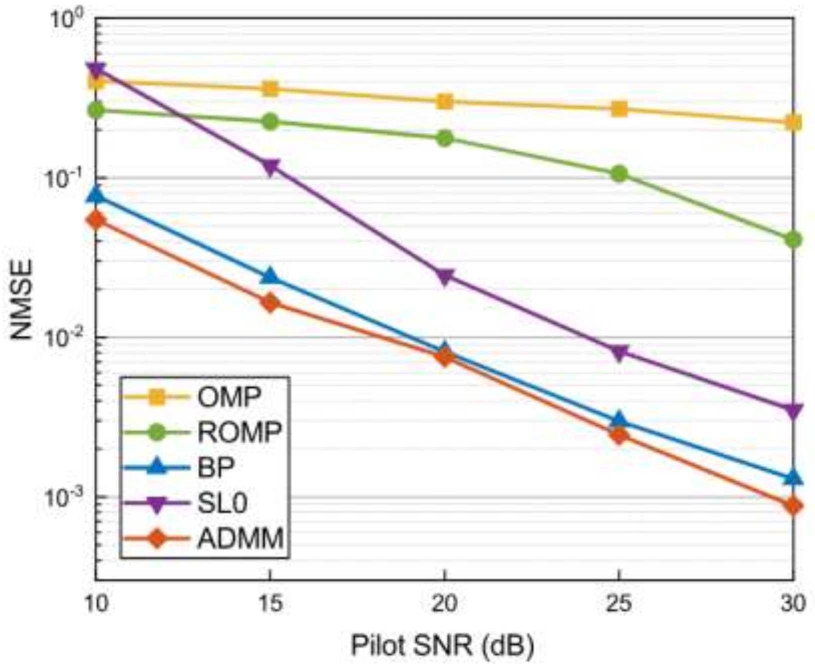
- ADMM iterations:
  - $\boldsymbol{h}^{(i+1)} \coloneqq \arg\min_{\boldsymbol{h}} \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{\Psi}\boldsymbol{h}\|_2^2 + \frac{\delta}{2}\left\|\boldsymbol{h} - \boldsymbol{b}^{(i)} + \boldsymbol{v}^{(i)}\right\|_2^2$
  - $\boldsymbol{b}^{(i+1)} \coloneqq \arg\min_{\boldsymbol{b}} \frac{\lambda}{\delta}\|\boldsymbol{b}\|_1 + \frac{1}{2}\left\|\boldsymbol{h}^{(i+1)} - \boldsymbol{v}^{(i)} + \boldsymbol{b}\right\|_2^2$
  - $\boldsymbol{v}^{(i+1)} \coloneqq \boldsymbol{v}^{(i)} + \boldsymbol{h}^{(i+1)} - \boldsymbol{b}^{(i+1)}$
- Soft-thresholding $S_\rho(\cdot)$ enforces sparsity while retaining dominant paths.

Simulation Results

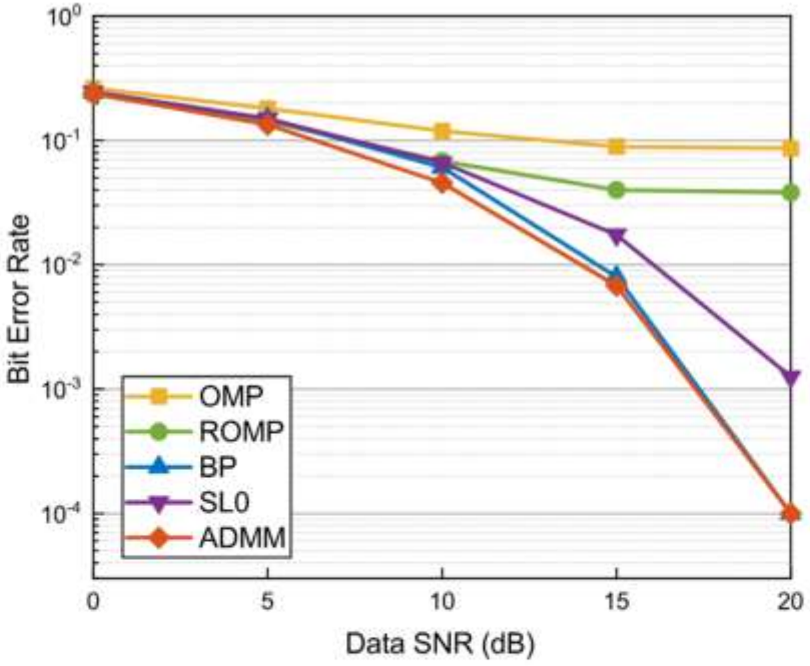
- ✓ **Simulation Results**
- ✓ **Complexity Comparison**

Simulation Results

- Setup: 4GHz, 15kHz spacing, 400 km/h (3GPP ETU), 9 paths, QPSK.
- Results:
  - ➢ NMSE vs. SNR



  - ➢ BER vs. SNR



Complexity Comparison

- Dominant cost: matrix inversion $\rightarrow \mathcal{O}\left((k_\nu l_\tau)^3\right)$,
  feasible as $k_\nu \ll N, l_\tau \ll M$.

- Comparison:
  - OMP – Fast, inaccurate
  - SL0 – Non-convex, unstable
  - BP – Accurate, slow
  - ADMM – Convex, efficient, robust

Algorithm	Average Time (s)
OMP	0.332×10^{-2}
ROMP	4.630×10^{-2}
BP	382.2×10^{-2}
SL0	0.349×10^{-2}
ADMM	5.258×10^{-2}

(Monte Carlo experiments on device of Intel (R) Core (TM) i5-12400F CPU and 32GB of RAM)

Conclusions

- ✓ **Conclusions**
- ✓ **Limitations**

Conclusions

- **Conclusions:**
  - ➢ ADMM-based OTFS channel estimation → accurate, robust.
  - ➢ Better NMSE and BER performance without sparsity prior than other CS methods.

- **Limitations:**
  - ➢ Relatively high complexity → 2D structure
  - ➢ Fractional Doppler, Practical waveforms not considered.

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Thanks!