Low-Complexity AFDM Signal Detection for High-Mobility and LEO Satellite Communications

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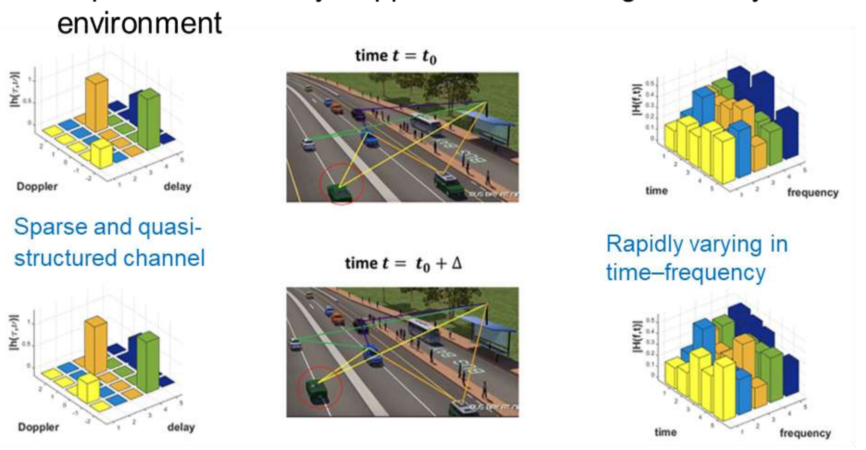
$\log q_i(z_i) \leftarrow \mathbb{E}_{q_{-i}}[\log p(\mathbf{x},\mathbf{z})] + \text{const}$

Abstract

High-mobility and LEO satellite channels cause severe Doppler dispersion that degrades OFDM performance. Affine frequency division multiplexing (AFDM) mitigates this effect with an efficient chirp-based design. To further lower receiver complexity, this paper introduces a variational Bayesian (VB)detector using mean-field factorization and coordinate-ascent updates, eliminating matrix inversion. The proposed method achieves fast, low-complexity soft detection with superior BER for next-generation LEO systems.

Background

- **Orthogonal Time Frequence Space (OTFS)**
  - Exploitation of delay-doppler domain in high-mobility environment



  - Significant taps are few and well-localized, so receivers can explicitly exploit **delay–Doppler diversity** (multiple distinct paths) for coding and detection ➔ 2D-spreading in TF

$$y[l,k] \approx \sum_{\ell=1}^{L} \alpha_\ell \sum_{l'=0}^{M-1}\sum_{k'=0}^{N-1} x[l',k']\, g_M(l-l'-l_\ell-\delta_\ell)\, g_N(k-k'-k_\ell-\kappa_\ell)$$

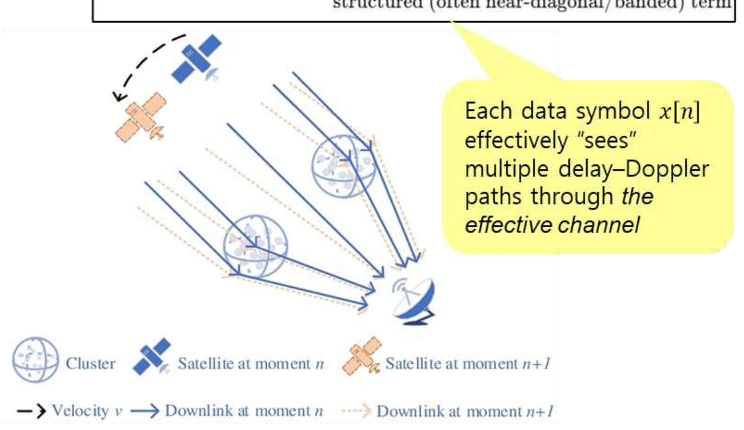
  - The near-structured sparse matrices allow **low-complexity equalizers** (e.g., message passing, banded solvers)

- **Affine Frequency Division Multiplexing (AFDM)**
  - Another modulation schemes for doubly selective (time-varying + frequency-selective) channels
  - *Chirp/DAFT mixing + path combining* in a 1D basis

$$\mathbf{H} \approx \sum_{\ell=1}^{L} \alpha_\ell \mathbf{\Pi}^{l_\ell}\mathbf{\Delta}(\nu_\ell)$$

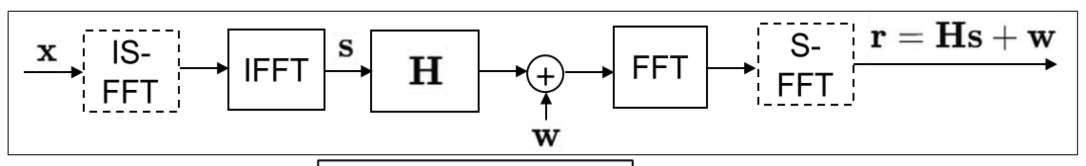
$$\mathbf{y} = \mathbf{A}_{\text{AFDM}}\mathbf{H}\mathbf{A}^H_{\text{AFDM}}\mathbf{x} + \mathbf{w}$$

$$\mathbf{H}_{\text{eff,AFDM}} = \sum_{\ell=1}^{L} \alpha_\ell \underbrace{\mathbf{A}_{\text{AFDM}}\mathbf{\Pi}^{l_\ell}\mathbf{\Delta}(\nu_\ell)\mathbf{A}^H_{\text{AFDM}}}_{\text{structured (often near-diagonal/banded) term}}$$



System Model: OTFS vs. AFDM

- **Input-output Model**



Transmit time-domain vector: $\mathbf{s} \in \mathbb{C}^{N\times 1}$
Receive time-domain vector: $\mathbf{r} \in \mathbb{C}^{N\times 1}$
Channel matrix in time domain: $\mathbf{H} \in \mathbb{C}^{N\times N}$
Noise: $\mathbf{w} \in \mathbb{C}^{N\times 1}$

- Channel model: $\mathbf{H} = \sum_{\ell=1}^{L} \alpha_\ell \mathbf{\Pi}^{l_\ell}\mathbf{\Delta}(\nu_\ell)$ with $\mathbf{\Delta}(\nu_\ell) = \text{diag}\left(e^{j2\pi\nu_\ell n}\right)_{n=0}^{N-1}$

Sums of delay-shift and Doppler-phase operators

- **OTFS:** Orthgonal Time Frequency Space
  - OTFS modulation: $\mathbf{s} = \underbrace{\mathbf{F}^H_{\text{TF}}\mathbf{U}_{\text{ISFFT}}}_{\mathbf{A}_{\text{OTFS}}}\mathbf{x}_{\text{DD}}$ — *2D (Delay-Doppler) ➔ 2D modulation*
  - OTFS demodulation: $\mathbf{y}_{\text{DD}} = \mathbf{A}^H_{\text{OTFS}}\mathbf{r}$
  - Received signal: *Physical delay-Doppler*

$$\mathbf{y}_{\text{DD}} = \mathbf{A}^H_{\text{OTFS}}\mathbf{H}\mathbf{A}_{\text{OTFS}}\mathbf{x}_{\text{DD}} + \mathbf{A}^H_{\text{OTFS}}\mathbf{w}$$

$$\mathbf{y}_{\text{DD}} = \mathbf{H}_{\text{DD}}\mathbf{x}_{\text{DD}} + \mathbf{w}_{\text{DD}}, \quad \mathbf{H}_{\text{DD}} = \mathbf{A}^H_{\text{OTFS}}\mathbf{H}\mathbf{A}_{\text{OTFS}}$$

For ideal pulses and integer Doppler: $\mathbf{H}_{\text{DD}}$ is sparse (few paths). With fractional Doppler/practical pulses: $\mathbf{H}_{\text{DD}}$ becomes dense but structured (Dirichlet leakage); OTFS is matched to doubly selective channels, but detection can be harder.

- **AFDM:** Affine Frequency Division Multiplexing
  - Discrete-time affine FT (DAFT): $\mathbf{A} = \mathbf{\Lambda}_{c_2}\mathbf{F}\mathbf{\Lambda}_{c_1}$, $\mathbf{\Lambda}_c = \text{diag}\left(e^{-j2\pi c n^2}\right)_{n=0}^{N-1}$ — *Diagonal chirp matrix*
  - AFDM modulation: $\mathbf{s} = \mathbf{A}^H\mathbf{x}$, $\mathbf{x} \in \mathbb{C}^N$
  - AFDM demodulation: $\mathbf{y} = \mathbf{A}\mathbf{r}$. — *1D vector ➔ 1D modulation*
  - Received signal

$$\mathbf{y} = \mathbf{A}\mathbf{r} = \mathbf{A}(\mathbf{H}\mathbf{s}+\mathbf{w}) = \mathbf{A}\mathbf{H}(\mathbf{A}^H\mathbf{x}) + \mathbf{A}\mathbf{w}$$

$$\mathbf{y} = \underbrace{\mathbf{A}\mathbf{H}\mathbf{A}^H}_{\mathbf{H}_{\text{eff,AFDM}}}\mathbf{x} + \mathbf{w}_{\text{AFDM}}$$

DAFT basis is designed to diagonalize or localize delay–Doppler channel operators, yielding a structured effective channel.

- **ML Detection for Unified Model**
  - Linear model: $\mathbf{y} = \mathbf{H}_{\text{eff}}\mathbf{x} + \mathbf{w}$
  - ML detection: $\hat{\mathbf{x}}_{\text{ML}} = \arg\min_{\mathbf{x}\in\mathcal{A}^K} \|\mathbf{y} - \mathbf{H}_{\text{eff}}\mathbf{x}\|^2$
  - Complexity of brute-force ML: $\mathcal{O}(|\mathcal{A}|^K \cdot K^2)$
  - Exponential in $K = MN$ and becomes intractable quickly (e.g., 16^{256} is impossible)

- **Detection for OTFS**
  - Each observation mixes many symbols:

$$y_i = \sum_{j=1}^{K} H_{ij}x_j + w_i$$

➔ Near-ML detection is challenging mainly because the effective channel is often **highly coupled**, making approximate inference more expensive

➔ Per-iteration cost roughly $\mathcal{O}(nnz(\mathbf{H}))$ or $\mathcal{O}(K^2)$ if dense

- **Detection for AFDM**
  - AFDM is designed such that the transformed effective channel $\mathbf{H}_{\text{eff,AFDM}}$ is often more structured (near-diagonal or banded) under delay–Doppler channels

$$y_i = h_{ii}x_i + w_i \Rightarrow \hat{x}_i = \arg\min_{a\in\mathcal{A}} |y_i - h_{ii}a|^2$$

  - If diagonal, complexity $\mathcal{O}(K|\mathcal{A}|)$ ➔ linear in K
  - If banded with bandwidth B, i.e., each symbol couples to about B neighbors, many iterative methods become $\mathcal{O}(KB)$ per iteration

Variational Inference: Overview

- **Aim:** To find the best approximation within a family of distributions $\mathcal{Q}$ such as $q(\mathbf{z}) \in \mathcal{Q}$

$$q(\mathbf{z}) \approx p(\mathbf{z} \mid \mathbf{x})$$

  - Find the member of tractable family of distributions that is closest to the true posterior, by solving an optimization problem: $p(\mathbf{z}\mid\mathbf{x}) \approx q^\star(\mathbf{z}), \quad q^\star = \arg\max_{q\in\mathcal{Q}} \text{ELBO}(q)$

- **Actual Construction and Computation of the Approximation**
  - Step 1: Choose a variational family $\mathcal{Q}$ ➔ $q(\mathbf{z}) = \prod_{i=1}^{K} q_i(z_i)$ *Mean-field family*
  - Step 2: Define the optimization objective ➔ $\text{ELBO}(q) = \mathbb{E}_q[\log p(\mathbf{x},\mathbf{z})] - \mathbb{E}_q[\log q(\mathbf{z})]$

$p(\mathbf{x},\mathbf{z}) = p(\mathbf{x}\mid\mathbf{z})p(\mathbf{z})$ →

$$\text{ELBO}(q) = \underbrace{\mathbb{E}_q[\log p(\mathbf{x}\mid\mathbf{z})]}_{\text{data consistency}} - \underbrace{D_{\text{KL}}(q(\mathbf{z})\|p(\mathbf{z}))}_{\text{regularization}}$$

i.e., this objective balances data consistency and regularization

  - Step 3: compute $q^\star$ by optimization ➔ Coordinate Ascent Variational Inference (CAVI)

$$\log q_i(z_i) \leftarrow \mathbb{E}_{q_{-i}}[\log p(\mathbf{x},\mathbf{z})] + \text{const}$$

Hold all other factors fixed, update q_i using the expected log joint distribution, iterate until convergence

CAVI for Receiver Design

- **Actual Construction and Computation of the Approximation**
  - Step 1: Choose a variational family $\mathcal{Q}$ ➔ $q(\mathbf{z}) = \prod_{i=1}^{K} q_i(z_i)$ *: Mean-field family*
  - Step 2: Define the optimization objective ➔ $\text{ELBO}(q) = \mathbb{E}_q[\log p(\mathbf{x},\mathbf{z})] - \mathbb{E}_q[\log q(\mathbf{z})]$

$p(\mathbf{x},\mathbf{z}) = p(\mathbf{x}\mid\mathbf{z})p(\mathbf{z})$ → $\text{ELBO}(q) = \underbrace{\mathbb{E}_q[\log p(\mathbf{x}\mid\mathbf{z})]}_{\text{data consistency}} - \underbrace{D_{\text{KL}}(q(\mathbf{z})\|p(\mathbf{z}))}_{\text{regularization}}$

  - Step 3: compute $q^\star$ by optimization ➔ Coordinate Ascent Variational Inference (CAVI)

$\log q_i(z_i) \leftarrow \mathbb{E}_{q_{-i}}[\log p(\mathbf{x},\mathbf{z})] + \text{const}$ *Hold all other factors fixed, update q_i using the expected log joint distribution, iterate until convergence*

- **Turning the CAVI Rule into an Implementable Detector**
  - Linear baseband model: $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \quad \mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$

$$\log p(\mathbf{y},\mathbf{x}) = -\frac{1}{\sigma^2}\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 + \log p(\mathbf{x})$$

$q_i(x_i) \propto \exp\left(\mathbb{E}_{q_{-i}}[\log p(\mathbf{y},\mathbf{x})]\right)$ →

$$q_i(a) \propto p_i(a)\exp\left(-\frac{1}{\sigma^2}\|\mathbf{r}_i - \mathbf{h}_i a\|^2\right), \quad a \in \mathcal{A}.$$

$$\mathbf{r}_i = \mathbf{y} - \sum_{j\neq i}\mathbf{h}_j m_j \qquad m_j = \mathbb{E}_{q_j}[x_j] = \sum_{a\in\mathcal{A}} a\, q_j(a)$$

- **The Proposed VB Detector: Algorithm**

Algorithm 1 Variational Bayesian AFDM Detector

Inputs: Received signal $\mathbf{y}$, effective channel matrix $\bar{\mathbf{H}}$, noise variance N_0, modulation order K, max iterations N_{iter}, tolerance ϵ.

Initialization: $\boldsymbol{\mu}_n^{(0)} \leftarrow \mathbf{y}$, $\pi^{(0)} \leftarrow \mathbf{1}/K$, $\hat{\mathbf{x}}^{(0)} \leftarrow \bar{\mathbf{H}}^H(\bar{\mathbf{H}}\bar{\mathbf{H}}^H + N_0\mathbf{I})^{-1}\mathbf{y}$, $\hat{\mathbf{v}}^{(0)} \leftarrow \mathbf{1}$.

1: **for** $t = 1$ to N_{iter} **do**
2: **for** each symbol $j = 1$ to N **do**
Step 1: Compute the residual signal.
3: $\boldsymbol{\mu}_n^{(t)} \leftarrow \mathbf{y} - \sum_{n'\neq n}\mathbf{H}(:,n')\hat{\mathbf{x}}_{n'}^{(t-1)}$
Step 2: Derive the equivalent scalar model.
4: $\sigma_n^{2\,(t)} \leftarrow N_0 + \sum_{n'\neq n}\|\mathbf{H}(:,n')\|^2\hat{v}_{n'}^{(t-1)}$
5: $z_n^{(t)} \leftarrow \frac{\mathbf{H}(:,n)^H\boldsymbol{\mu}_n^{(t)}}{\|\mathbf{H}(:,n)\|^2}$
Step 3: Update the posterior distribution.
6: $\pi_n^{k\,(t)} \propto \exp\left(-\frac{|a_k - z_n^{(t)}|^2}{\sigma_n^{2\,(t)}}\right), \ \forall a_k \in \mathbb{A}$
7: $\hat{x}_n^{(t)} \leftarrow \sum_{k=1}^{Q}\pi_n^{k\,(t)}a_k$, $\hat{v}_n^{(t)} \leftarrow \sum_{k=1}^{Q}\pi_n^{k\,(t)}|a_k - \hat{x}_n^{(t)}|^2$
8: **end for**
9: **if** satisfies converge condition **then break**
10: **end for**

Output: Detected symbols $\hat{\mathbf{x}} \leftarrow \arg\max_{\mathbb{A}} \boldsymbol{\pi}$.

Mean interference cancellation: removes the *expected contribution* of all other symbols (based on their current soft estimates) from the received signal

This step computes a symbol-dependent effective noise variance by adding the weighted posterior variances of other symbols, thereby folding "residual interference uncertainty" into the noise model to enable stable and accurate variational detection.

Simulation Results and Conclusion

Simulation Results

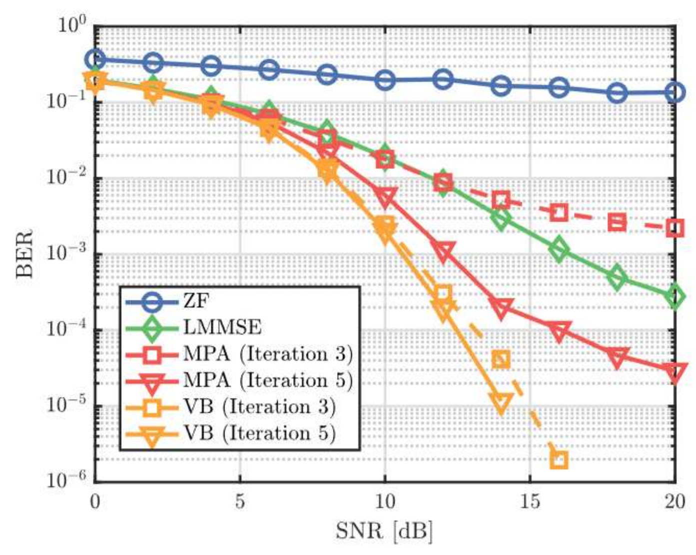


Fig. 1. The BER performance for $P = 3$ with different algorithms.

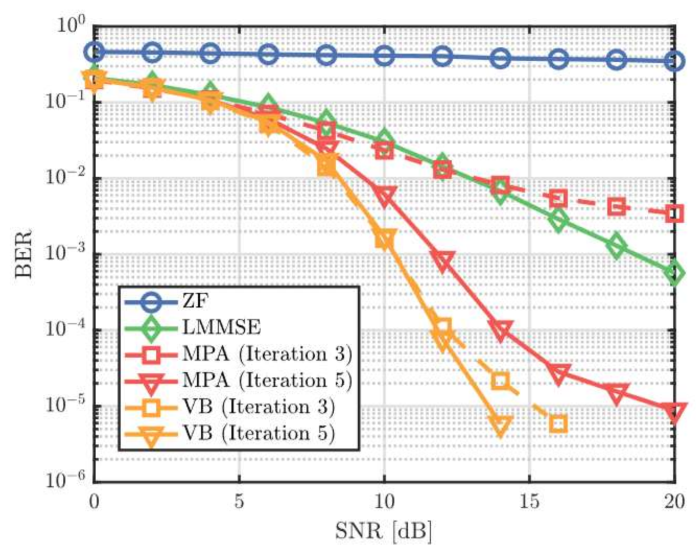


Fig. 2. The BER performance for $P = 5$ with different algorithms.

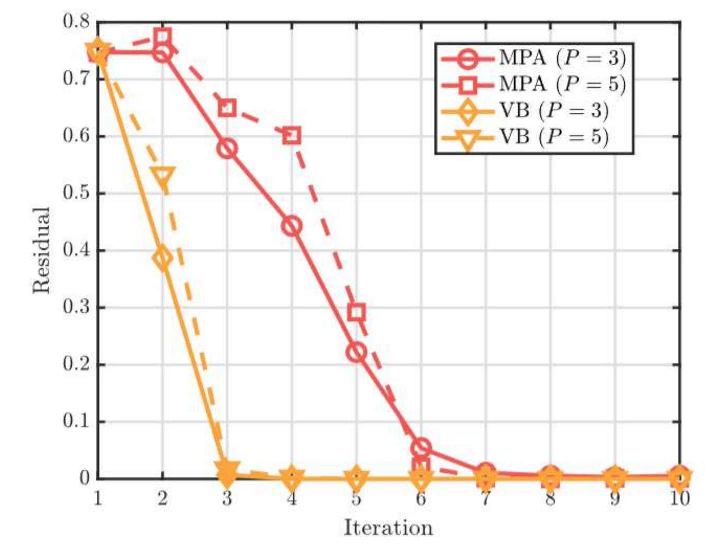


Fig. 3. Iteration-wise residuals of MPA and VB detector under different multipath numbers.

TABLE I
COMPARATION OF DOMINENT COMPLEXITY.

Algorithm	Complexity
MAP	Q^N
ZF	$\mathcal{O}(N^3)$
LMMSE	$< \mathcal{O}(N^3)$
MP	$\mathcal{O}(N_{\text{iter}}QNP^2)$
VB	$\mathcal{O}(N_{\text{iter}}QNP)$

- Performance gains become more pronounced as the number of paths increase.
- The VB detector consistently outperforms linear detectors
- Compared with MPA, the VB detector achieves comparable or improved BER with fewer iterations.
- The VB detector converges within a small number of iterations.

Conclusion

- VB detector enables low-complexity and robust symbol detection for AFDM in high-mobility channels.