



Delay-Doppler Domain Channel Estimation for OTFS System Using the Gradient Projection Method

Can Zheng, Chung G. Kang



School of Electrical Engineering, Korea University

Nov. 26, 2025

Outline

- **Introduction**
 - Background
 - Motivation
- **System Model**
 - OTFS Input-Output Relationship in Delay-Doppler Domain
 - Compressed Sensing-based Channel Estimation Model
- **Proposed GP-SOONE Algorithm**
- **Simulation Results**
 - Simulation Results
- **Summary**
 - Conclusions
 - Future Work

Outline

- **Introduction**
 - Background
 - Motivation
- **System Model**
 - OTFS Input-Output Relationship in Delay-Doppler Domain
 - Compressed Sensing-based Channel Estimation Model
- **Proposed GP-SOONE Algorithm**
- **Simulation Results**
 - Simulation Results
- **Summary**
 - Conclusions
 - Future Work

Background

New business mode



Satellite Networks



Vehicle-to-

Aug, 2025: 6G meeting settles on the same old air interface.

1 个月前 • 🌐

First agreement of 6G has been made!
CP-OFDM and DFT-s-OFDM will be used in 6G 🎉

Thanks to our feature lead [Karri Ranta-aho!](#)

显示译文



826

39 条评论 • 36 次转发

arios
tworks
inuum

for high-dynamic channels

e of smooth evolution

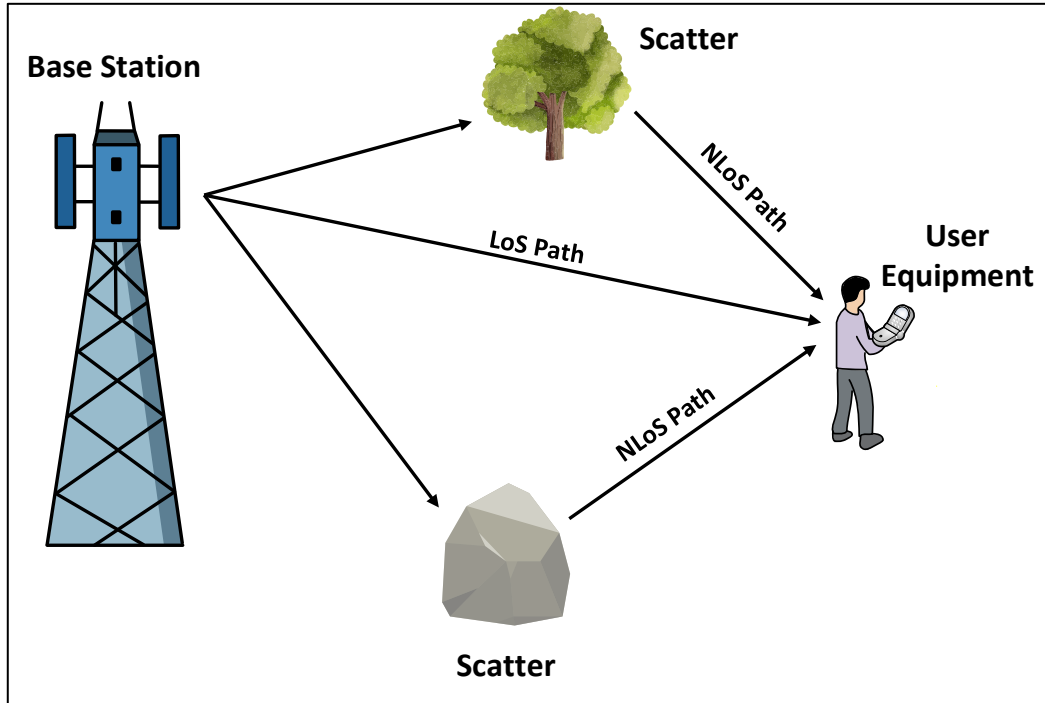
massive MIMO

table for integrated

ns

Motivation

Recall: How OFDM (multi-carrier) combats multipath?



Figure

"Time-domain spread is difficult to handle, so put the problem in the frequency domain, where it becomes simpler."

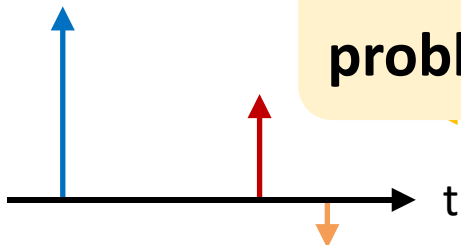


Figure: Delay profile (channel impulse response).

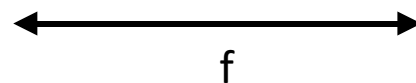


Figure: Frequency domain: selective fading.

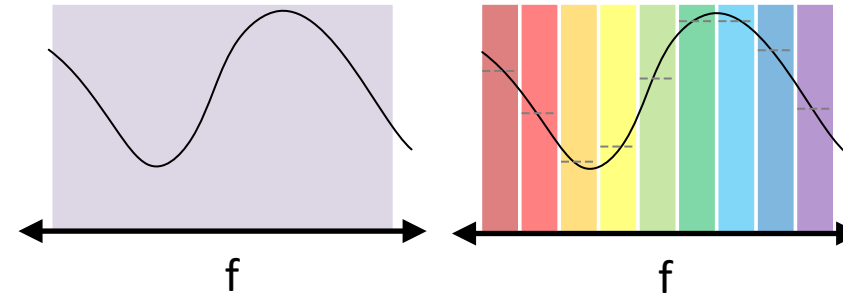


Figure: In frequency domain, multi-carrier system (right) slices the spectrum through using the subcarriers, while single-carrier system suffers from frequency-selective channel.



Problem

- Time-domain spread, Inter-symbol interference, difficult to handle in the time domain

Core idea of OFDM

- **Cyclic prefix:** time-domain convolution into a cyclic convolution.
- **Perform an FFT:** time-domain convolution into

- Each subcarrier sees a "flat channel".
- Multipath can be compensated **simply** by **one-tap equalization**.

Motivation

From the experience of "solving problems with one domain" to the motivation for OTFS

Core idea:

If the channel is selective only in one domain, we can shift the problem to another domain, where the channel is **flatter** and **easier to equalize**.

The problem with time-varying multipath channels:

- Selective in both time and frequency (Doubly-selective).
- No simple "one-dimensional transformation" that can make them completely flat in any domain.

The idea behind OTFS:

- Redefine the entire communication system in the **delay-Doppler (DD) domain**.

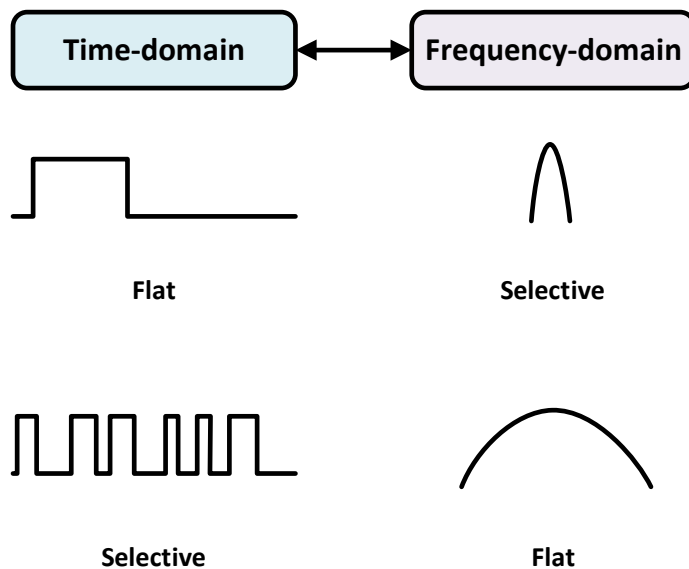


Figure: Spectral contents of a signal depend on its variations in time.

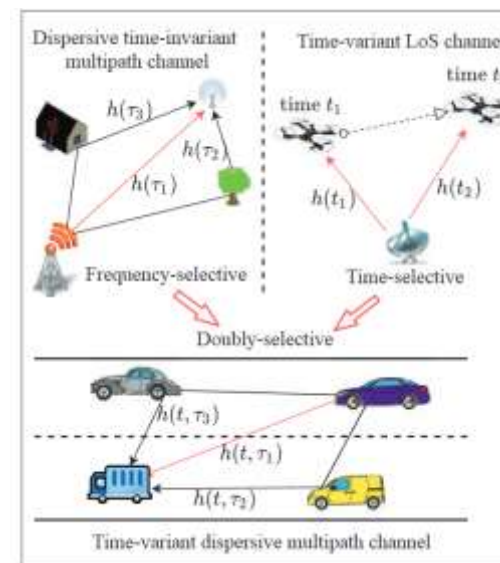


Figure: An illustration of frequency-selective, time-selective, and doubly-selective channel models.*

*Z. Wei *et al.*, "Orthogonal Time-Frequency Space Modulation: A Promising Next-Generation Waveform," in *IEEE Wireless Communications*, vol. 28, no. 4, pp. 136-144, August 2021.

Motivation

Why delay-Doppler domain?

- **Mathematical Significance: Rotational Invariance**
 - Time invariant channel: $y_k = H_k x_k$
 - Time-varying channel: $Y = H_{DD} * X$
- **Physical Significance**
 - Delay $\tau \leftrightarrow$ distance.
 - Doppler $\nu \leftrightarrow$ relative velocity
- **Key Advantages over Time-Frequency (TF) Domain**
 - Sparsity
 - Resolvability
 - Stability
 - Compactness

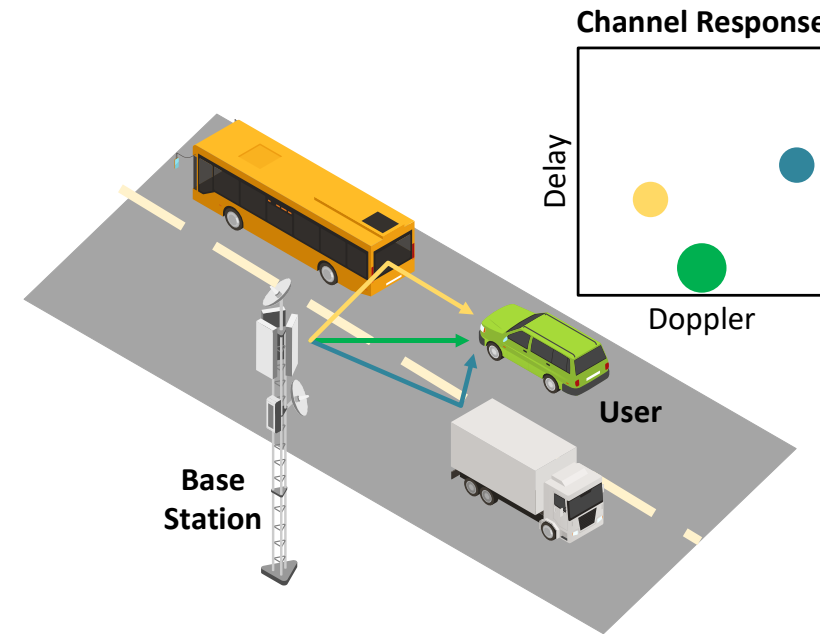
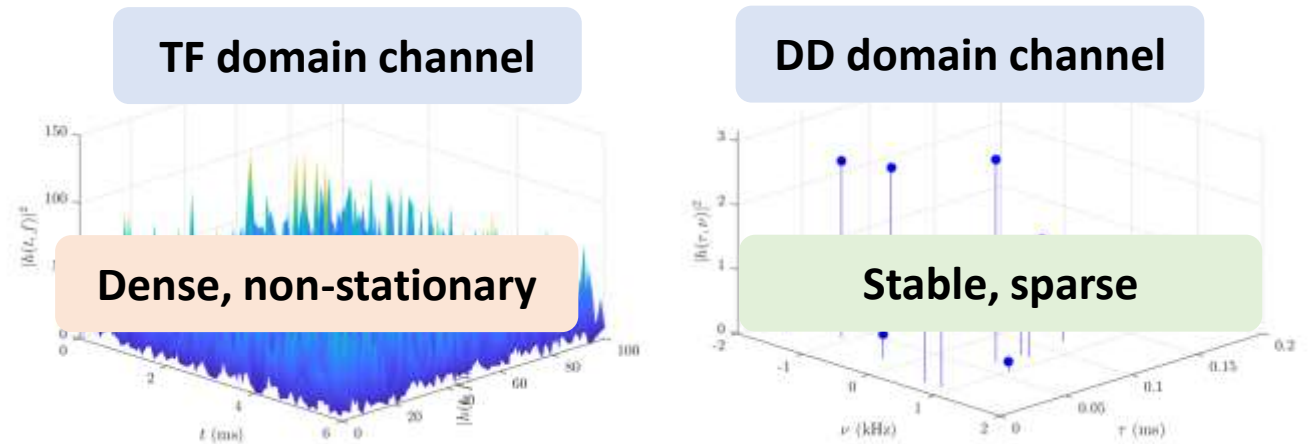


Figure: Wireless channel representation.



$$h(t, f) = \iint h(\tau, \nu) e^{-j(t\nu - \tau f)} d\tau d\nu$$

$$h(\tau, \nu) = \sum_{i=1}^P h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i)$$

* If the transmit and receive pulse shaping functions are perfectly localized in time and frequency, then they satisfy the bi-orthogonality condition.

Figure: The time-frequency vs delay-Doppler channel representation of a high-mobility multipath channel (linear time-varying)

Outline

- **Introduction**
 - Background
 - Motivation
- **System Model**
 - OTFS Input-Output Relationship in Delay-Doppler Domain
 - Compressed Sensing-based Channel Estimation Model
- **Proposed GP-SOONE Algorithm**
- **Performance Analysis**
 - Simulation Results
- **Summary**
 - Conclusions
 - Future Work

System Model

OTFS input-output relationship

OTFS modulation:

- DD domain-modulated signal $x[k, l]$.
- **Inverse Symplectic finite Fourier transform (ISFFT):** DD domain to TF domain, $x[k, l] \rightarrow X[n, m]$.
- **Heisenberg Transform:** TF domain to time series, $X[n, m] \rightarrow s(t)$.

OTFS demodulation:

- Time domain received signal $r(t)$.
- **Wigner Transform:** matched filter, $r(t) \rightarrow Y[n, m]$.
- **SFFT:** TF domain to DD domain, $Y[n, m] \rightarrow y[k, l]$.

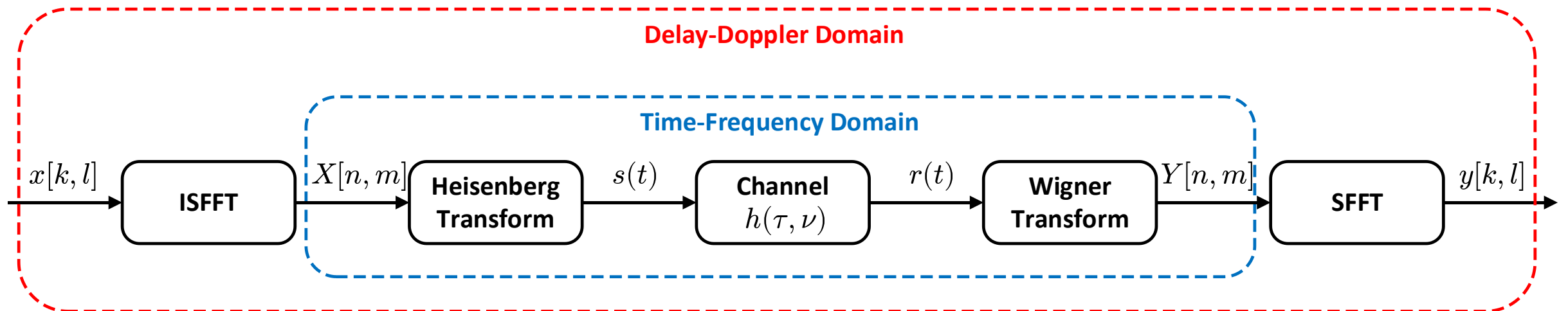


Figure: OTFS modulation and demodulation process.

System Model

- Compressed sensing (CS)-based channel estimation model

- The end-to-end input-out relationship:

$$y_p[k', l'] = \frac{1}{NM} \sum_{k=-k_v}^{k_v} \sum_{l=0}^{l_\tau} x[k' - k, l' - l] \cdot \bar{h}[k, l] + n[k', l'],$$

$$\mathbf{y} = \Psi \mathbf{h} + \mathbf{w}$$

Index mapping:

$$\mathbf{y} \left(l'k_p + k' + \frac{k_p}{2} + 1 \right) = y_p[k', l'],$$

$$\Psi \left(l'k_p + k' + \frac{k_p}{2} + 1 \right) = x[k' - k, l' - l],$$

$$\mathbf{h}((2l + 1)k_v + k + 1) = \bar{h}[k, l]$$

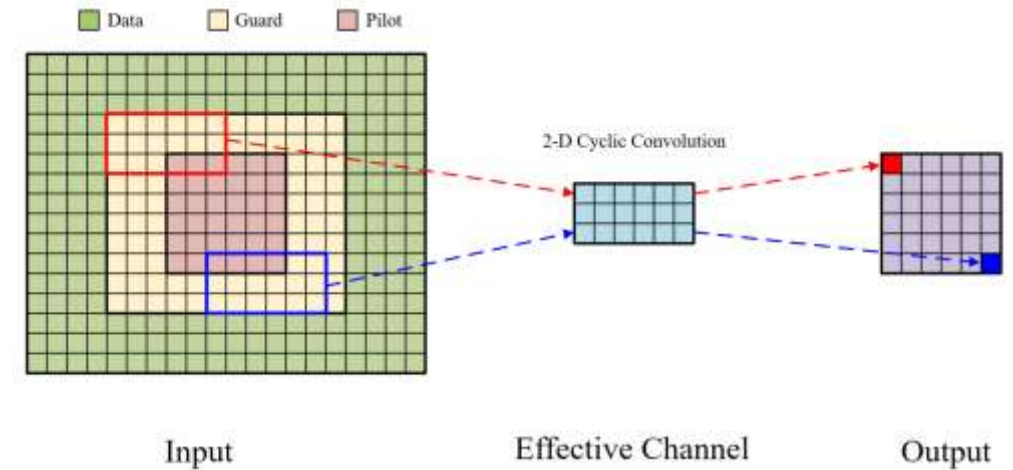


Figure: OTFS modulation and demodulation process.

Formulate as a **sparse recovery problem**:

$$\min_{\mathbf{h}} \|\mathbf{h}\|_0 \quad \text{s.t.} \quad \|\mathbf{y} - \Psi \mathbf{h}\|_2 < \varepsilon,$$

where $\|\mathbf{h}\|_0 = \sum_{k=1}^K 1_{\{h_k \neq 0\}}$, and ε is a nonnegative small number.

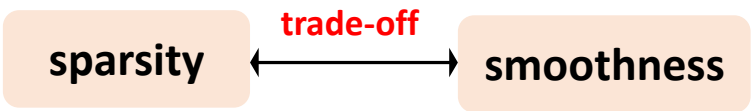
NP-hard!

Outline

- **Introduction**
 - Background
 - Motivation
- **System Model**
 - OTFS Input-Output Relationship in Delay-Doppler Domain
 - Compressed Sensing-based Channel Estimation Model
- **Proposed GP-SOONE Algorithm**
- **Performance Analysis**
 - Simulation Results
- **Summary**
 - Conclusions
 - Future Work

GP-SOONE: gradient projection with sequential one-order negative exponential

Proposed GP-SOONE Algorithm



Previous Work: Idea of SLO-based channel estimation*

- Using a **smooth** function that "looks like" l_0 -norm.
- Use gradient descent
- Estimate the channel

Essential Problem to solve

- **SLO:** Transform the l_0 -norm minimization problem into:

$$\min_{\mathbf{h}} F_{\sigma}(\mathbf{h}) \quad s.t. \quad \|\mathbf{y} - \Psi\mathbf{h}\|_2 < \varepsilon,$$

where $F_{\sigma}(\mathbf{h}) \propto \|\mathbf{h}\|_2^2$.

- **GP-SOONE:** Transform the l_0 -norm minimization problem into:

$$\min_{\mathbf{h}} G_{\sigma}(\mathbf{h}) \quad s.t. \quad \|\mathbf{y} - \Psi\mathbf{h}\|_2 < \varepsilon,$$

where $G_{\sigma}(\mathbf{h}) \propto \|\mathbf{h}\|_1$.

the approximation increasingly closer to the true l_0 -norm;

- ④ Obtain sparse solution.

Problem:

More essentially l_0 than "sparsity".

more biased

method:

on, choose a large
im parameters;

gradient descent

(under the constraint $\mathbf{y} \approx \Psi\mathbf{h}$);

- ③ Gradually decrease σ ;
- ④ Obtain sparse solution.

* X. Wang, C. Zheng, P. Hu, J. Yang and C. G. Kang, "A Sparsity-Agnostic SLO Channel Estimation Approach for OTFS Systems," in *IEEE Communications Letters*, vol. 29, no. 5, pp. 1097-1101, May 2025.

Proposed GP-SOONE Algorithm

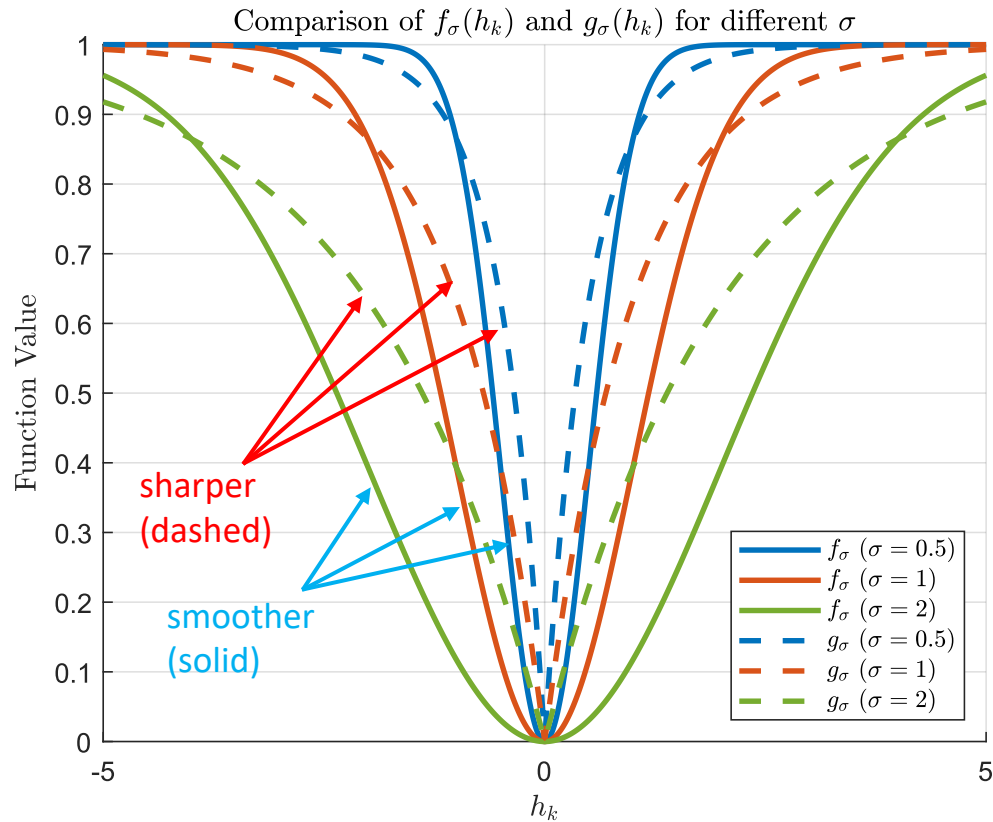


Figure: Comparison of f_σ and g_σ for different σ .

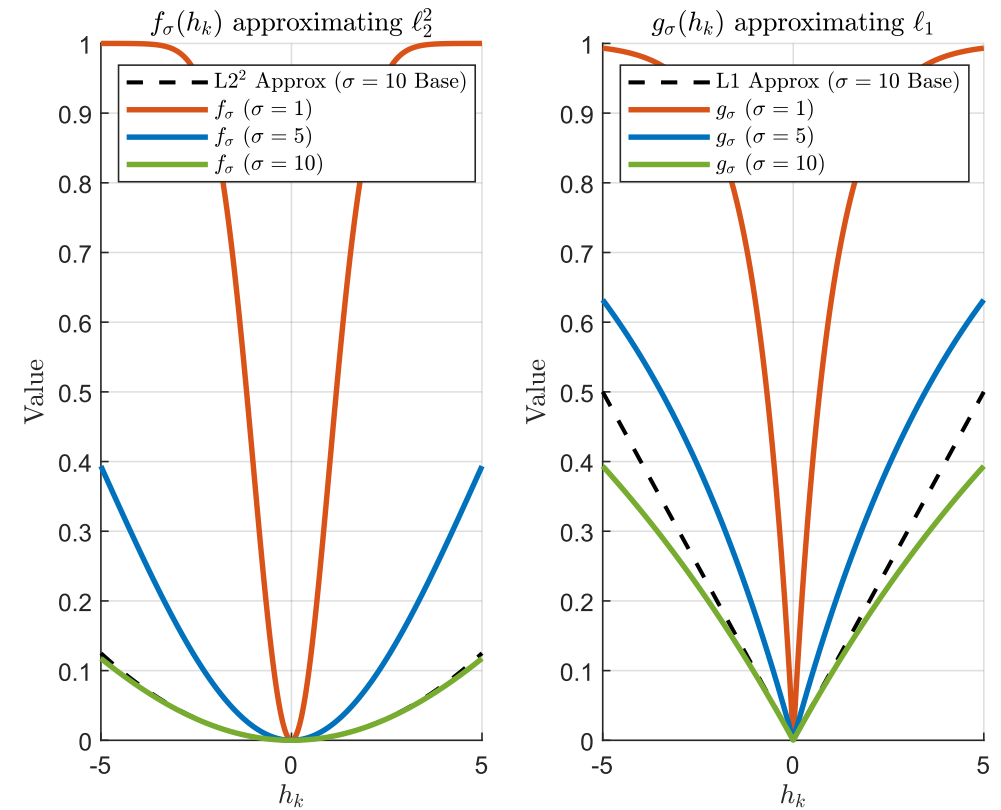


Figure: L1 and L2 norm approximation of f_σ and g_σ .

Proposed GP-SOONE Algorithm

Algorithm explanation and complexity analysis

Algorithm 1 The GP-SOONE Algorithm

Inputs: Pilot matrix Ψ and its pseudo-inverse Ψ^\dagger , received signal \mathbf{y} , minimum kernel width σ_{\min} , decay factor γ , step parameters L_0, L_1 , inner loop iterations L .

Initialization: $\hat{\mathbf{h}}^{(0)} = \Psi^\dagger \mathbf{y}$, $\sigma^{(0)} = 100 \cdot \max(|\hat{\mathbf{h}}^{(0)}|)$, \longrightarrow Initialization

$N_{\text{iter}} = \left\lceil \frac{\log(\sigma^{(0)}/\sigma_{\min})}{\log(1/\gamma)} \right\rceil$ \longrightarrow Number of outer iteration

1: **for** $j = 1$ to N_{iter} **do** \longrightarrow **Outer iteration:** First find a general direction at the cost of "smoothing approximation", and then gradually reduce σ to approach the true sparse solution.

2: $\sigma^{(j)} \leftarrow \gamma \cdot \sigma^{(j-1)}$ \longrightarrow Update kernel width

3: $\alpha^{(j)} \leftarrow \frac{N_{\text{iter}} - \frac{j}{2} + 1}{N_{\text{iter}}}$ \longrightarrow Update outer step size

4: $\mathbf{h} \leftarrow \hat{\mathbf{h}}^{(j-1)}$

5: **for** $l = 1$ to L **do** \longrightarrow **Inner iteration:** Perform gradient projection updates under the current σ .

6: $\beta^{(l)} \leftarrow \frac{L - \frac{l}{2} + 1}{L}$ \longrightarrow Inner step size weight

7: $\mu^{(j,l)} \leftarrow \alpha^{(j)} \cdot \beta^{(l)} \cdot \min\left(\frac{\max(|\mathbf{h}|)}{L_0}, \frac{\sigma^{(j)}}{L_1}\right)$ \longrightarrow Adaptive step size \longrightarrow

8: $\delta \leftarrow \text{sign}(\mathbf{h}) \odot \exp\left(-\frac{|\mathbf{h}|}{\sigma^{(j)}}\right)$ \longrightarrow Gradient direction of sparse penalty

9: $\mathbf{h} \leftarrow \mathbf{h} - \mu^{(j,l)} \cdot \delta - \Psi^\dagger(\Psi\mathbf{h} - \mathbf{y})$ \longrightarrow Update \mathbf{h}

10: **if** $\|\mathbf{y} - \Psi\mathbf{h}\|_2 < \epsilon$ **then** \longrightarrow Check convergence

11: **break**

12: **end if**

13: **end for**

14: $\hat{\mathbf{h}}^{(j)} \leftarrow \mathbf{h}$

15: **end for**

16: **Output:** Estimated channel vector $\hat{\mathbf{h}} = \hat{\mathbf{h}}^{(N_{\text{iter}})}$

Algorithm	Complexity
OMP	$O(P^3J + PJK)$
SAMP	$O(PJ^3)$
SLO	$O(J^2 + N_{\text{iter}}LJK)$
GP-SOONE	$O(J^2 + N_{\text{iter}}LJK)$

- α : continuous approximation strategy
- β : ensure the inner loop gradually converges
- $\frac{\max|\mathbf{h}|}{L_0}$: numerical stability
- $\frac{\sigma}{L_1}$: Ensure that optimization remains stable

Outline

- **Introduction**
 - Background
 - Motivation
- **System Model**
 - OTFS Input-Output Relationship in Delay-Doppler Domain
 - Compressed Sensing-based Channel Estimation Model
- **Proposed GP-SOONE Algorithm**
- **Performance Analysis**
 - Simulation Results
- **Summary**
 - Conclusions
 - Future Work

Performance Analysis

Parameter Settings

Parameter	Values
Carrier frequency (f_c)	4 GHz
Sub-carrier spacing (Δf)	15 kHz
Maximum speed (v_{\max})	400 km/h
Maximum delay (τ_{\max})	5000 ns
Number of paths (P)	9
Modulation mode	QPSK
Pilot Sequence	Zadoff-Chu Sequence
Size of a OTFS frame (M, N)	(32, 64)
Pilot length (l_p, k_p)	(6, 10)
Guard intervals (l_τ, k_ν)	(5, 9)
Step parameters L_0, L_1	25, 10

Simulation Result: SNR vs. NMSE

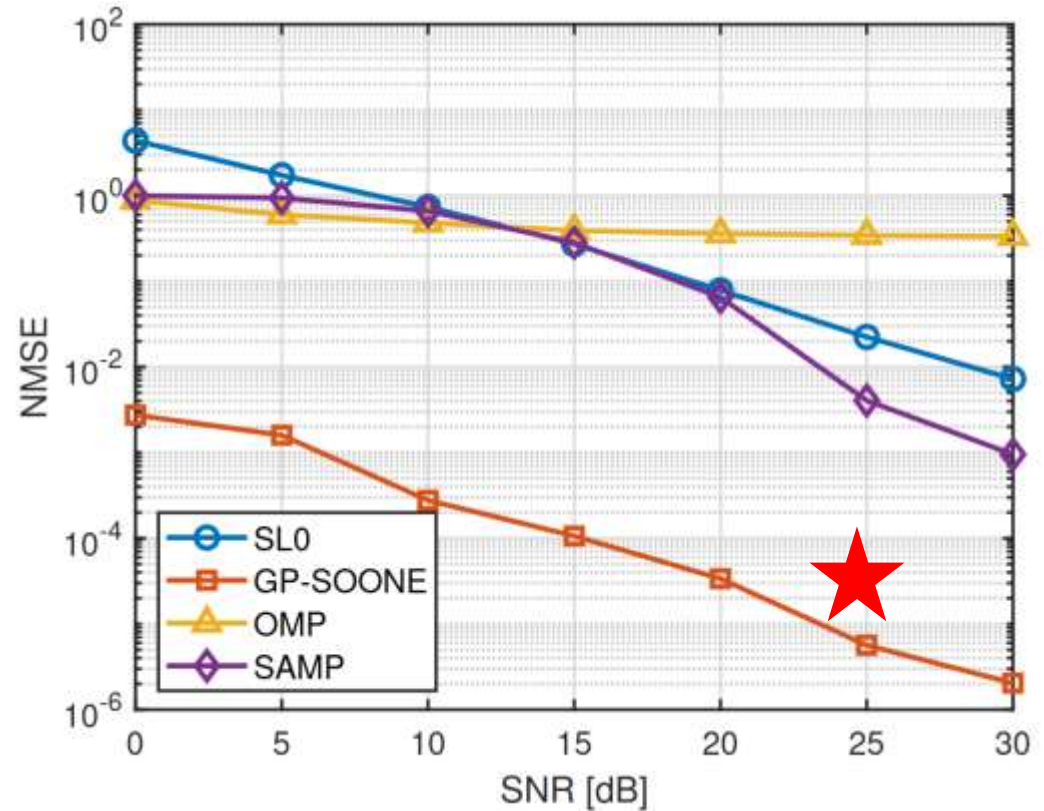


Figure: The NMSE performance comparison against the SNR.

Outline

- **Introduction**
 - Background
 - Motivation
- **System Model**
 - OTFS Input-Output Relationship in Delay-Doppler Domain
 - Compressed Sensing-based Channel Estimation Model
- **Proposed GP-SOONE Algorithm**
- **Performance Analysis**
 - Simulation Results
- **Summary**
 - Conclusions
 - Future Work

Summary

Contribution

- **Proposed GP-SOONE method:**
 - Replaces the Gaussian kernel with a first-order negative exponential kernel for better approximation of l_0 -norm.
 - Requires no prior sparsity.
 - Employs a gradient projection structure, balancing data consistency and sparsity.
- **Simulation results:**
 - In integer Doppler OTFS scenarios, the performance (NMSE) significantly outperforms OMP, SAMP, and SL0.
 - Improves estimation accuracy while maintaining low complexity.

Limitations

- Simplified model: We assume bi-orthogonality condition is satisfied, and only integer-Doppler case considered.
- SISO-OTFS: How to extend it to MIMO-OTFS?

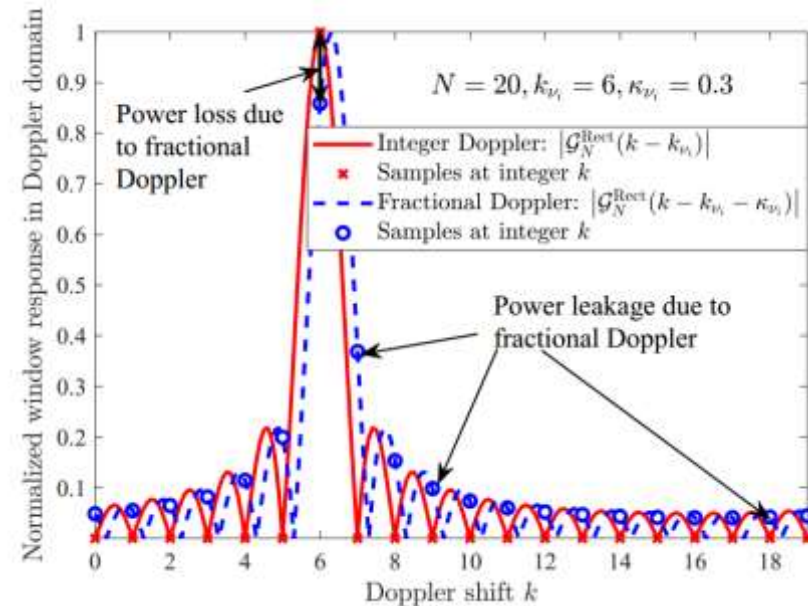


Figure: Illustration of fractional Doppler effect in the Doppler domain*.